Ratio estimator for the Sightability Model

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1 Introduction

This vignette documents the development of the ratio estimator in the *SightabilityModel* package (Fieberg, 2012).

The theory for this extension is found in Wong (1996). A portion of her thesis is appended to this document

2 Implementation

Equation 2.4.2 is the ratio estimator formed as the ratio of estimates of the total for variables a and b (e.g. bulls:cow ratio) that are adjusted using the inverse of sightability (Θ_{ij}) for group j in primary sampling unit i.

The delta-method is used to find the variance of the ratio as a function of the variances of the estimates of the numerator and denominator and the covariance between the two estimates (Equation 2.4.4). The variance of the numerator and denominator can be obtained using the *Sight.Est()* function in the *SightabilityModel* package. So that all remains is to estimate the covariance between the estimate of the numerator and denominator ($Cov(\widehat{\tau}_a, \widehat{\tau}_b)$)).

The first Equation 2.4.5 gives the formula for the covariance of the numerator and denominator. In order to estimate the covariance, we need estimators for each term, i.e.

$$\widehat{Cov}(\widehat{\tau}_a, \widehat{\tau}_b) = estimator\left(E[\widehat{\tau}_a\widehat{\tau}_b]\right) - estimator\left(\tau_a\tau_b\right)$$

The $estimator\left(E[\widehat{\tau}_a\widehat{\tau}_b]\right)$ is simply $\widehat{\tau}_a\widehat{\tau}_b$.

This then gives:

$$\widehat{Cov}(\widehat{\tau}_a, \widehat{\tau}_b) = \widehat{\tau}_a\widehat{\tau}_b - \widehat{\tau_a\tau_b}$$

where $\widehat{\tau_a\tau_b}$ is the estimator of $\tau_a\tau_b$. NOTICE THE MINUS SIGN between the two terms.

The population value for the second term above ($\tau_a\tau_b$) is given by the equation between the two Equations 2.4.5 which consists of three terms.

This is combined in the second Equation 2.4.5 and Equation 2.4.6. Note that the equation has an error. where the minus sign above has been dropped, so the proper form of the two estimating equations is:

$$\widehat{Cov}(\widehat{\tau}_a, \widehat{\tau}_b) = \widehat{\tau}_a\widehat{\tau}_b - [term2 + term3 + term4]$$

rather than simply adding the 4 terms together.

This corrections was validated using simulation.

3 References

Fieberg, J. (2012). Estimating Population Abundance Using Sightability Models: R SightabilityModel Package. Journal of Statistical Software, 51(9), 1-20. URL https://doi.org/10.18637/jss.v051.i09

Wong, C. (1996). Population size estimation using the modified Horvitz-Thompson estimator with estimated sighting probabilities. Dissertation, Colorado State University, Fort Collins, USA. Available from ProQuest Dissertations & Theses A&I. (304301226). Retrieved from http://proxy.lib.sfu.ca/login?url=https://search-proquest-com.proxy.lib.sfu.ca/docview/304301226?accountid=13800